Worcester County Mathematics League

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1

1. Following the proper order of operations, evaluate:

$$5(5+5) - 5 \cdot 5 \div 5 + 5$$

2. Let $a*b = \dfrac{a-b}{a}$ and $a\,\hat{}\,b = \dfrac{a}{b} + \dfrac{b}{a}$. Find the value of the following expression as a fraction reduced to lowest terms:

$$(4\,\hat{}\,5) \cdot \left[(4*5) + 3\right]^{-1}$$

3. Following the proper order of operations, evaluate:

$$\left(1 - \frac{92}{189}\right) \cdot \left(\frac{76 + 6\left(\dfrac{13}{3} - \dfrac{5}{2}\right) - 81}{\dfrac{\dfrac{1}{12} \cdot 16\frac{4}{7}}{\dfrac{3}{2} \cdot 8\frac{2}{7}} \div \dfrac{\dfrac{1}{4} \cdot 8\frac{1}{6}}{\dfrac{7}{12} \cdot 16\frac{1}{6}}} \right)$$

ANSWERS

(1 pt.)   1._____

(2 pts.)   2._____

(3 pts.)   3._____

Douglas, Bancroft, Tahanto

Worcester County Mathematics League
Freshman Meet 1 – November 3, 2010
Round 2: Solving Linear Equations

△2

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

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1. Solve for x: $\quad \dfrac{1}{4}\left(x - \dfrac{1}{3}\right) - \dfrac{1}{6}(x - 3) = \dfrac{2}{3}$

2. Solve for y: $\quad 0.3(2.5y - 0.5) = 0.6(1.5y + 0.3)$

3. Solve for z: $\quad -17(7z - 83) + 28(7z - 83) - 34 = 12(7z - 83)$

ANSWERS

(1 pt.)   1._____

(2 pts.)   2._____

(3 pts.)   3._____

Worcester County Mathematics League
Freshman Meet 1 – November 3, 2010
Round 3: Logic Problems

All answers must be placed in the answer section at the bottom

NO CALCULATOR ALLOWED

1. Mr. Foley and Mr. Harris are playing a game in which the winner of a round receives 25 cents from the loser. Mr. Foley has won four rounds and Mr. Harris has $1.25 more than he had at the beginning. Find the least number of rounds that could have been played.

2. On a balance scale, three green balls balance six red balls; two yellow balls balance five red balls; and six red balls balance four white balls. How many red balls are needed to balance four green, two yellow and two white balls?

3. The last names of Holly, John, Lilly and Jack are: Hobbie, Doe, White and O'Lantern (in no particular order). Doe, who is O'Lantern's grandfather, is not related to Holly. John is 5 years old. John is not related to Jack or White. What is Lilly's last name?

ANSWERS

(1 pt.)   1. _____ rounds

(2 pts.)   2. _____ red balls

(3 pts.)   3. _____

Worcester County Mathematics League

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

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1. You always ride your bike at the constant speed of 15 miles per hour. How many miles do you travel in 5 minutes?

2. When air is pumped into a tire, the pressure required varies inversely as the volume of air. If the pressure is 30 lbs / in^2 when the volume is 140 in^3, find the pressure (in lbs / in^2) when the volume is 100 in^3.

3. For this problem, assume that the number of males and the number of females in the nation's population are equal. According to a national survey, there are twice as many left-handed males as there are left-handed females. If 15% of the total population is left-handed, compute the ratio of the number of right-handed males to the number of right-handed females. Please write your answer as a fraction reduced to lowest terms.

ANSWERS

(1 pt.)   1._____ miles

(2 pts.)   2._____ lbs / in^2

(3 pts.)   3._____

Worcester County Mathematics League
Freshman Meet 1 – November 3, 2010
TEAM ROUND

All answers must *either* be in <u>simplest exact form</u> *or* as <u>decimals rounded</u> <u>correctly to at least three decimal places</u>　　　(3 pts. each)

<u>**APPROVED CALCULATORS ALLOWED**</u>

1. How many positive factors of 20,100 are even?

2. Chris traveled 1 hour longer and 2 miles farther than Calvin, but averaged 3 miles per hour slower. If the sum of their travel times was 4 hours, what was the sum (in miles) of the distances that they traveled?

3. Mahogany weighs 33.94 pounds per cubic foot, whereas pine weighs 23.45 pounds per cubic foot. What is the total weight (in pounds) of a mahogany board measuring $5\frac{1}{2}$ inches by 1 inch by 6 feet and a pine board measuring $3\frac{1}{2}$ inches by $1\frac{1}{2}$ inches by 8 feet?

4. The digits in base-16 are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. Write 54321_{10} as a base-16 number. (Note that the subscript indicates the number's base)

5. I have a bag of marbles containing more than 20 marbles but less than 100 marbles. If I take them out 2 at a time, 1 marble is left over. If I take them out 3 at a time, 2 are left over. If I take them out 4 at a time, 3 are left over. Finally, if I take them out 5 at a time, 1 marble is left over. How many marbles are in the bag?

6. If x, y, and z are positive integers with $x + 2y + 2z = 2011$ and $2x + 2y + z = 2010$, find the smallest possible value of $x + y + z$.

7. A zoologist tagged 52 deer in a local forest and released them. Several weeks later 28 deer are caught (in a friendly way) and 8 of them have the zoologist's tag. Based on a proportion, how many deer should the zoologist expect to live in the forest?

8. Consider all of the arrangements of the letters of WOCOMAL with either the O's next to each other or the O's on the ends of the arrangement. For example, ACOOMEL and OWCMALO are two of these arrangements. Find the fraction of all of the possible arrangements that satisfies these conditions. Please give your answer as a fraction reduced to lowest terms.

Hudson, QSC, Sutton, Worcester Academy (4 &5), Auburn (6 & 7), Algonquin

Worcester County Mathematics League
Freshman Meet 1 – November 3, 2010
ANSWERS

<u>Round 1</u>
1. 50

2. $\dfrac{41}{55}$ (only)

3. 6

<u>Round 2</u>
1. 3

2. $-2.2 = -2\frac{1}{5} = -\dfrac{11}{5}$

3. 7

<u>Round 3</u>
1. 13

2. 16

3. O'Lantern

<u>Round 4</u>
1. $1.25 = 1\frac{1}{4} = \dfrac{5}{4}$

2. 42

3. $\dfrac{8}{9}$ (only)

<u>Team Round</u>

1. 24

2. $30.5 = 30\frac{1}{2} = \dfrac{61}{2}$

3. $14.6175 = 14\frac{247}{400} = \dfrac{5847}{400}$

≈ 14.618
(accept any of the above)

4. D431

5. 71

6. 1006

7. 182

8. $\dfrac{1}{3}$ (only)